education equals

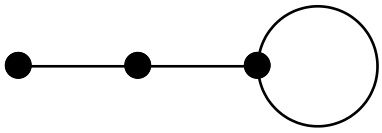
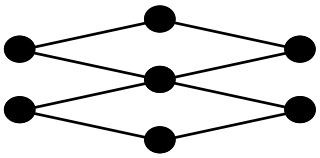
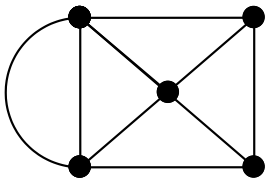
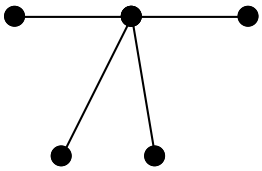
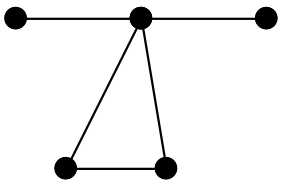
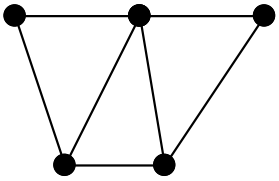
Topic: Eulerian and Hamiltonian graphs

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [9 marks]

Classify the following graphs as Eulerian (E), semi-Eulerian (SE), Hamiltonian (H), semi Hamiltonian (SH) or no classification (N).

Graph	Classification(s)
	
	
	
	
	
	

Question Two: [2, 2, 2, 2 : 8 marks]

Decide whether or not the following graphs are possible and if so illustrate an example.

a) a bipartite Eulerian graph

b) a complete bipartite semi Hamiltonian graph

c) a bipartite semi Eulerian graph

d) a bipartite Hamitonian and semi-Eulerian graph

Question Three: [2, 2, 2: 6 marks]

In a netball competition there are four teams participating.

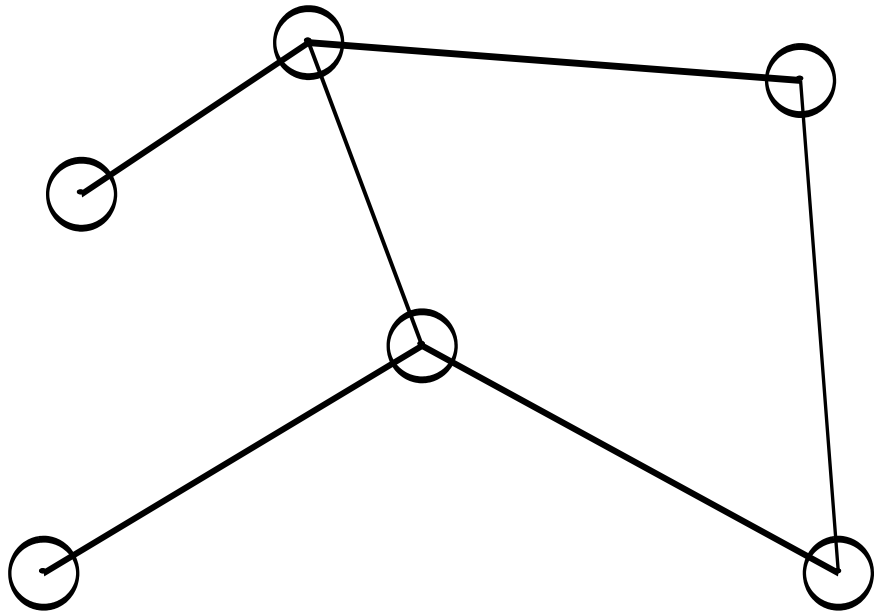
a) If every team needs to play each other exactly once, how many games are required for the season?

b) If a graph is created showing how each team can play each other, what type of graph would this be?

c) If two more teams join the competition how many more games are required to be played in order for each team to still play each other exactly once?

Question Four: [1, 1: 2 marks]

A treasure hunt is being organised and a graph showing where prizes are hidden is shown below.



a) Show a semi-Hamiltonian cycle which you could walk in order to collect all the prizes.

b) Is it possible to create a Eulerian cycle for the treasure hunt by adding exactly one path? If so, show the path on the graph above.

Question Five: [2, 2, 1, 1, 1, 2: 9 marks]

A tabloid magazine is comparing the amount of time some famous mothers spend on washing, cleaning and shopping per week.

Marg spends 3 hours on washing, 5 hours shopping and 9 hours cleaning each week. Bec spends, 6 hours on washing, 6 hours shopping and 3 hours cleaning each week and Kate spends 2 hours washing, 3 hours shopping and 4 hours cleaning each week.

a) Complete the following matrix showing the time (in hours) spent by each of the three mothers on these tasks each week. The rows in order represent the three mothers, Marg, Bec and Kate. The columns in order represent the tasks, washing, shopping and cleaning.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

b) Present the matrix from part a) as a labelled and weighted bipartite graph.

c) Is this bipartite graph complete or incomplete? Explain.

d) Who spends the most time cleaning?

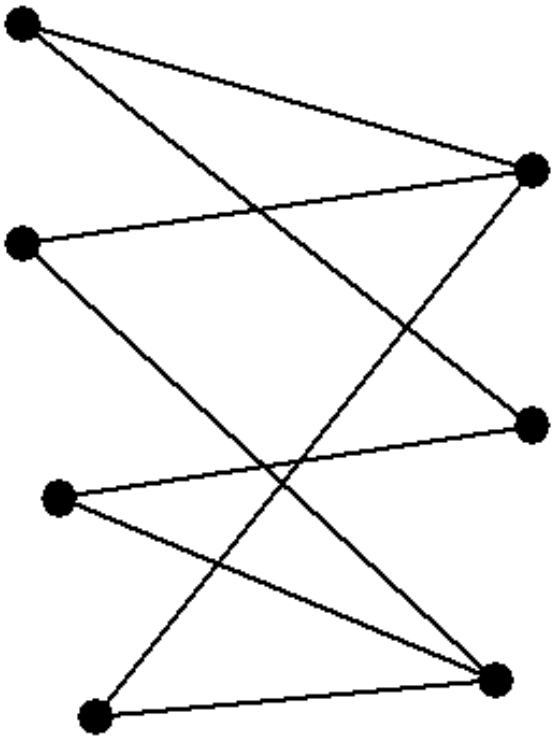
e) What is the total amount of time Bec spends on all these tasks?

f) Are there any Eulerian, Hamiltonian, semi-Eulerian or semi-Hamiltonian paths? If so clearly mark just one on your graph in part b). If not justify your answer.

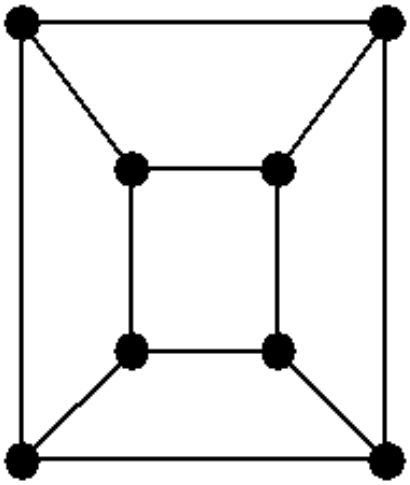
Question Six: [7 marks]

Decide which of the following graphs are classified as bipartite graphs. Those which are bipartite clearly label which vertices belong in each group.

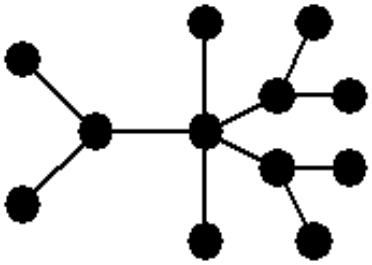
a)



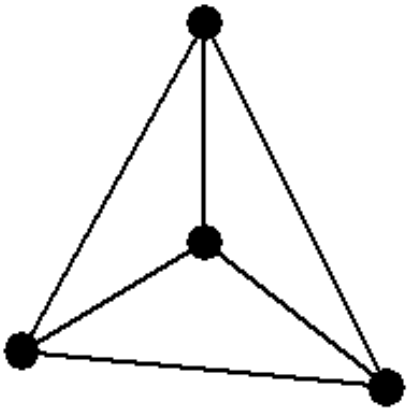
b)



c)



d)



Question Seven: [4 marks]

It is Valentine's Day and three women, Carly, Nicky and Becky will each receive gifts from their partners. The gifts they will receive are either roses, chocolates and/or cards. Each girl receives a card. Carly and Nicky both receive flowers and Nicky and Becky both receive chocolates.

Draw a graph to represent this situation and describe what type of graph it is.

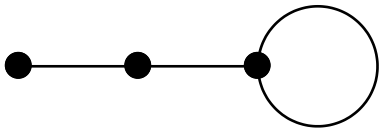
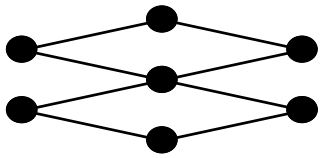
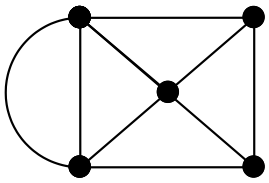
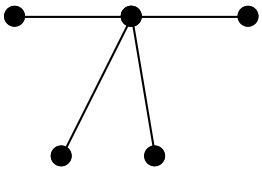
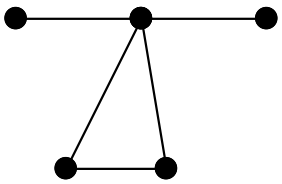
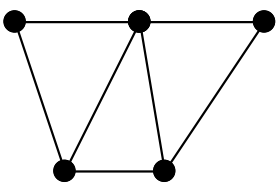
Topic: Eulerian and Hamiltonian graphs SOLUTIONS

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [9 marks]

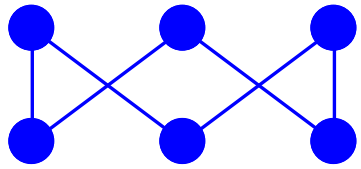
Classify the following graphs as Eulerian (E), semi-Eulerian (SE), Hamiltonian (H), semi Hamiltonian (SH) or no classification (N).

Graph	Classification(s)
	SE, SH ✓ ✓
	E ✓
	H, SE ✓ ✓
	N ✓
	SE ✓
	H, SE ✓ ✓

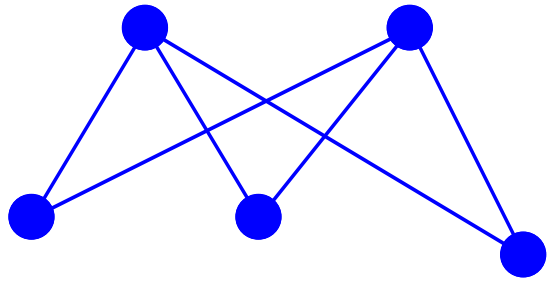
Question Two: [2, 2, 2, 2 : 8 marks]

Decide whether or not the following graphs are possible and if so illustrate an example.

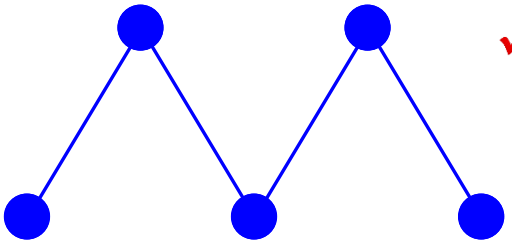
a) a bipartite Eulerian graph



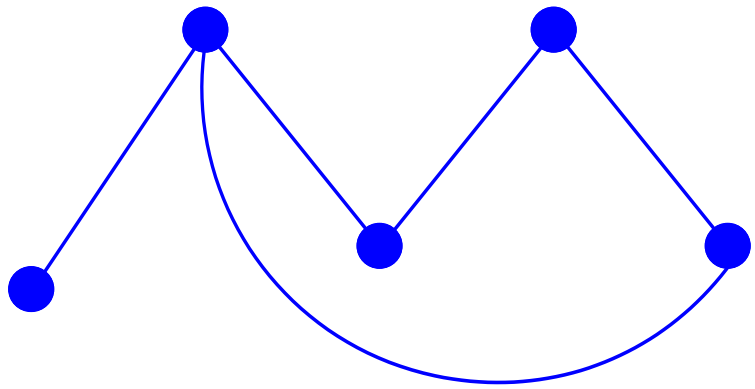
b) a complete bipartite semi Hamiltonian graph



c) a bipartite semi Eulerian graph



d) a bipartite Hamitonian and semi-Eulerian graph



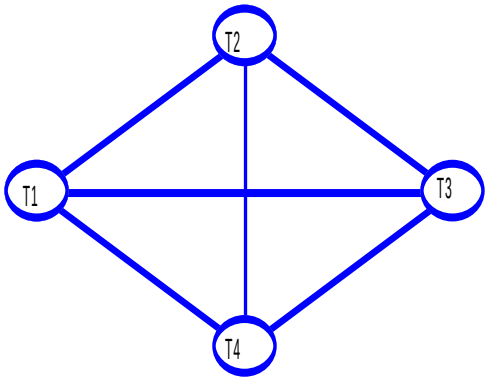
Question Three: [2, 2, 2: 6 marks]

In a netball competition there are four teams participating.

a) If every team needs to play each other exactly once, how many games are required for the season?

6 games are required ✓ ✓

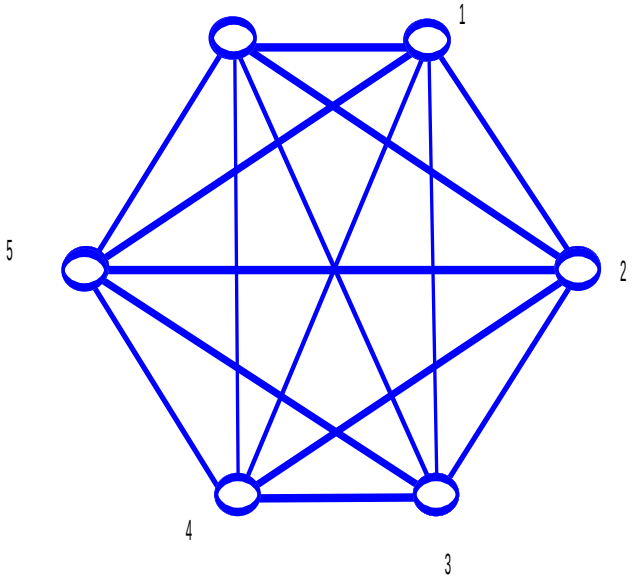
b) If a graph is created showing how each team can play each other, what type of graph would this be?



This is a complete graph because every vertex is connected by a single path to each other vertex in the graph.

✓ ✓

c) If two more teams join the competition how many more games are required to be played in order for each team to still play each other exactly once?

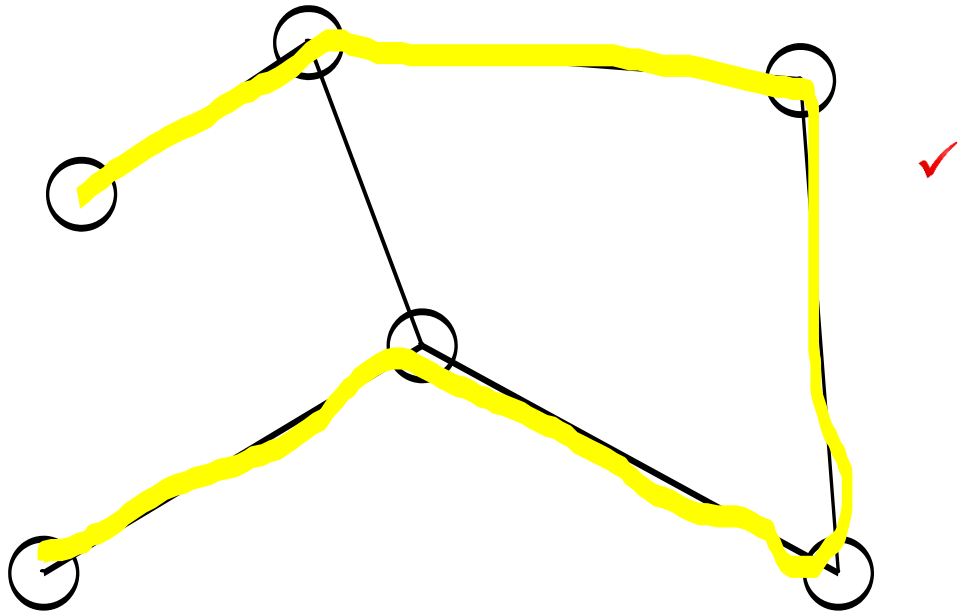


✓ ✓

$5 + 4 + 3 + 2 + 1 = 15$ ***games in total 6 played already so 9 more to play***.

Question Four: [1, 1: 2 marks]

A treasure hunt is being organized and a graph showing where prizes are hidden is shown below.



a) Show a semi-Hamiltonian cycle which you could walk in order to collect all the prizes.

b) Is it possible by adding exactly one path to create a Eulerian cycle for the treasure hunt? If so, show the path on the graph above.

No, you could make a semi-Eulerian graph by adding one path but not a Eulerian graph.

Question Five: [2, 2, 1, 1, 1, 2: 9 marks]

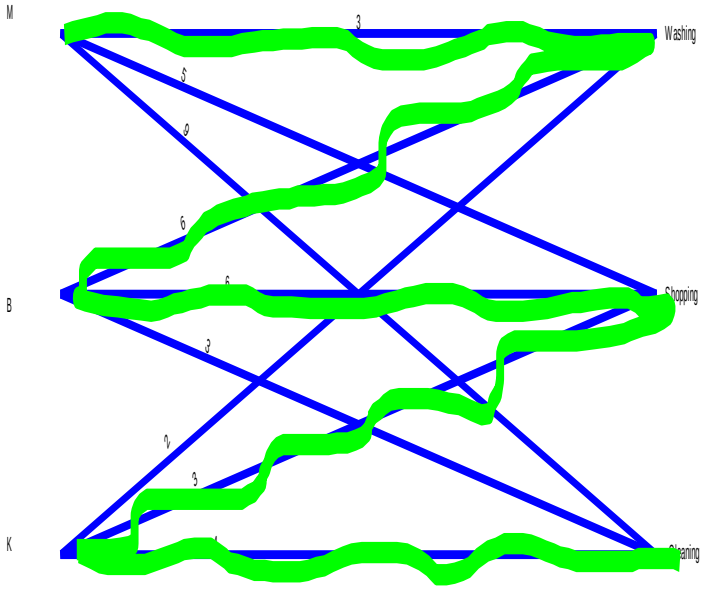
A tabloid magazine is comparing the amount of time some famous mothers spend on washing, cleaning and shopping per week.

Marg spends 3 hours on washing, 5 hours shopping and 9 hours cleaning each week. Bec spends, 6 hours on washing, 6 hours shopping and 3 hours cleaning each week and Kate spends 2 hours washing, 3 hours shopping and 4 hours cleaning each week.

a) Complete the following matrix showing the time (in hours) spent by each of the three mothers on these tasks each week. The rows in order represent the three mothers, Marg, Bec and Kate. The columns in order represent the tasks, washing, shopping and cleaning.

$$\begin{bmatrix} 3 & 5 & 9 \\ 6 & 6 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

b) Present the matrix from part a) as a labelled and weighted bipartite graph.



c) Is this bipartite graph complete or incomplete? Explain.

Yes it is a complete bipartite graph because every vertex of the first set is connected to every vertex of the second set.

d) Who spends the most time cleaning?

Marg

e) What is the total amount of time Bec spends on all these tasks?

15 hours

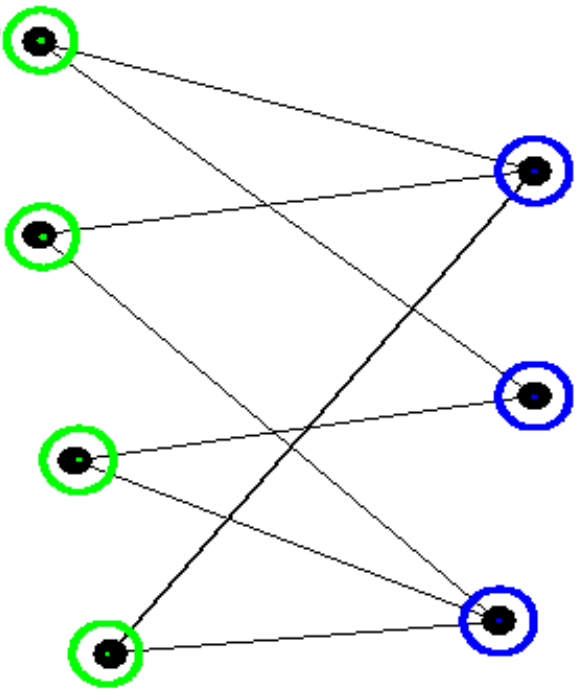
f) Are there any Eulerian, Hamiltonian, semi-Eulerian or semi-Hamiltonian paths? If so describe just one, if not justify your answer.

A semi-Hamiltonian paths is shown on the graph above.

Question Six: [7 marks]

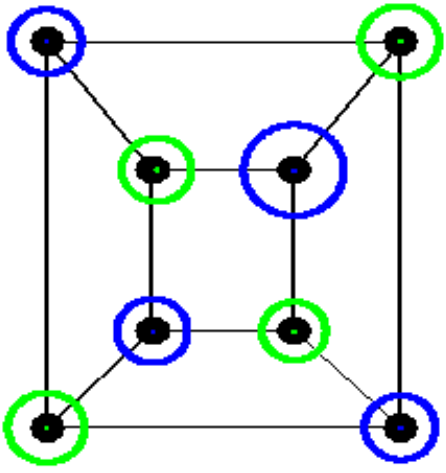
Decide which of the following graphs are classified as bipartite graphs. Those which are bipartite clearly label which vertices belong in each group.

a)



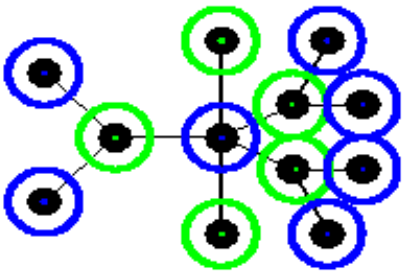
✓ ✓

b)



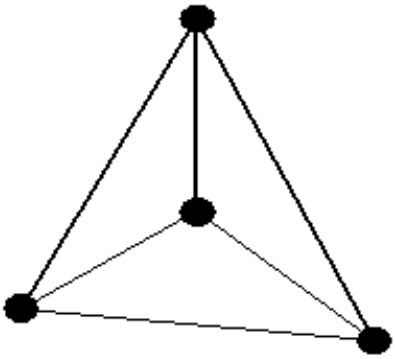
✓ ✓

c)



✓ ✓

d)



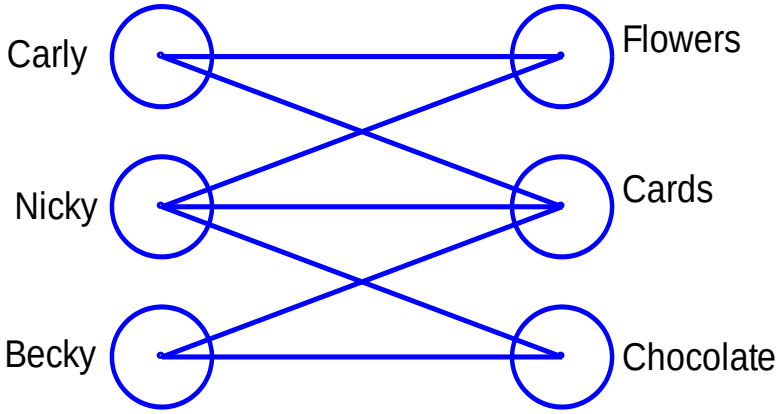
✓

Not biapartite

Question Seven: [4 marks]

It is Valentine's Day and three women, Carly, Nicky and Becky will each receive gifts from their partners. The gifts they will receive are either roses, chocolates and/or cards. Each girl receives a card. Carly and Nicky both receive flowers and Nicky and Becky both receive chocolates.

Draw a graph to represent this situation and describe what type of graph it is.



This is a bipartite graph. It has a semi-Eulerian and a semi-Hamaltonian path.